EXERCISE 1	DETERMINATION OF MOMENT OF INERTIA OF PHYSICAL PENDULUM AND VERIFICATION OF PARALLEL AXIS THEOREM

Measurement procedure

1. List of equipment
- Stand (a metal frame) with a metal rod for suspending the investigated pendulum
- Perforated disc
- Metal ring
- Scales
- Timer
- Caliper

2. Goals
- Experimental verification of the parallel axis theorem
- Experimental determination of the moment of inertia of a perforated disc and metal ring around different pivot points

3. Measurement setup

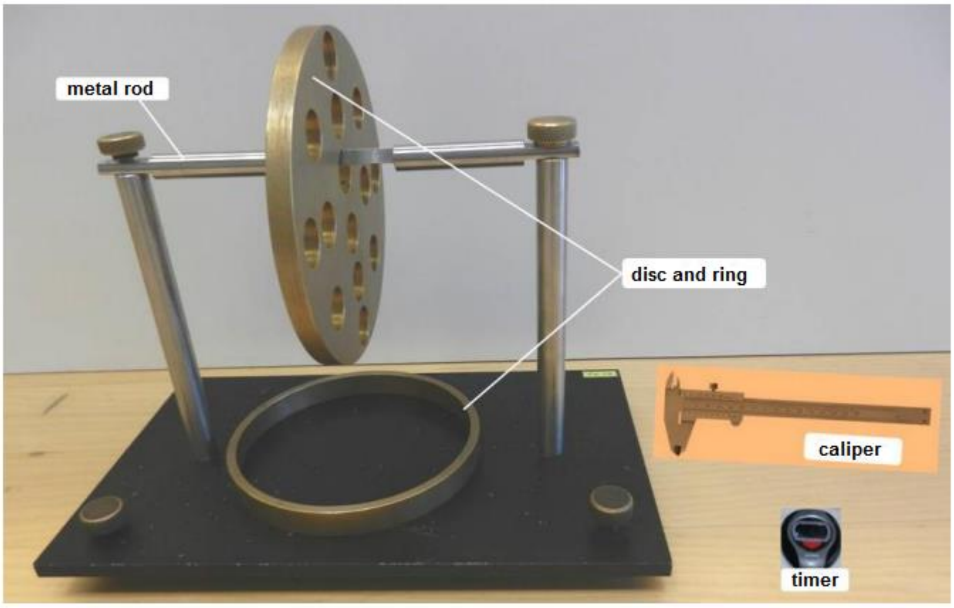

Fig. 1. The experimental setup used for determination of the moment of inertia. The metal rod is used as a pivot point for the physical pendulum. In the case of the perforated disc we can choose several pivot points located at different distances from the center of the disc.

4. Measurements

It is the supervisor who recommends which elements should be used in the experiment.

Part A - Perforated disc

a) Measure the mass *m* of the perforated disc using a scale.

b) Measure the doubled distance D_i between the pivot point and the center of mass (cf. Fig.2). For each chosen pair of holes repeat the measurements several times (approximately 6-7 times, unless the supervisor indicates otherwise).

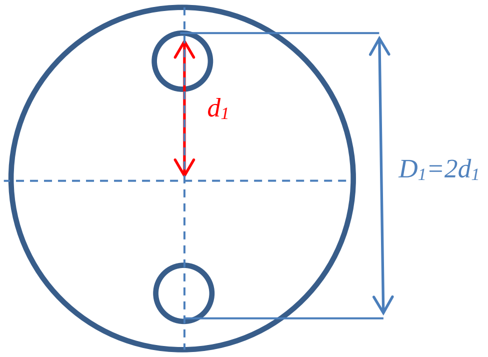

Fig.2. Diagram illustrating how to estimate the distance between holes.

c) Choose a pivot point and suspend the perforated disc on the metal rod. Measure the time of 100 oscillations of the pendulum. Keep in mind that the pendulum shouldn't be leant by an angle larger than a few degrees.

d) Repeat the oscillation time measurement 5 times.

Part B - Metal ring

a) Measure the mass *m* of the metal ring using a scale.

b) Using a caliper measure the internal *d* and external *D* diameter of the ring.

c) Using the metal frame, hang the ring on the rod so that it creates a physical pendulum. Measure the time of 100 oscillations of the ring around the pivot point. Repeat this measurement several times.

Part C – Metal bar

a) Measure the mass *m* of the metal bar using a scale.

b) Using a caliper measure the length l of the bar (repeat the measurement 6-7 times).

c) Measure the distance d_i between the pivot point and the center of mass (repeat the measurement 6-7 times).

d) Measure the time of 50 oscillations of the metal bar around the pivot point.

e) Repeat the oscillation time measurement 5 times.

f) Repeat instructions included in points 3-5 for another 4 pivot points.

5. Results analysis:

The supervisor decides whether the variant I or II should be used in data analysis.

Variant I

Part A (perforated disc):

a) Estimate uncertainties $u(D_1)$, $u(D_2)$, $u(D_3)$.... for the repeated measurements of the doubled distances D_1, D_2, D_3...

b) Calculate the distance between the pivot point and the mass center $d_i = \frac{D_i}{2}$ and its uncertainty $u_c(d_i)$.

c) For each pivot point calculate the average time $\bar{t}$ of $n = 100$ oscillations. Calculate the uncertainty $u(\bar{t})$. It can be assumed that the human reaction time is approximately equal to 0.5s.

d) Calculate the oscillation period $T = \bar{t}/n$ and its uncertainty $u_c(T)$ (n - number of oscillations). Assume $u(n) = 1$. Confirm the pendulum's oscillation period dependency on the moment of inertia.

e) Using the formula presented below calculate the moment of inertia I_d and its uncertainty $u_c(I_d)$.

$$I_d = \frac{T^2 mgd}{4\pi^2}$$

Repeat calculations for each pivot point (i.e. for each distance d).

f) Verify the parallel axis theorem. In order to do this check if for each pivot point the expression $T^2 gd - 4\pi^2 d^2 = C$ is constant within the margin of estimated error.

g) Using the estimated value of C for each pivot point calculate moment of inertia around the central axis and its uncertainty. Use the following formula:

$$I_0 = \frac{m}{4\pi^2} C$$

h) Using parallel axis theorem calculate the moment of inertia around the central axis and its uncertainty. Calculations repeat for each pivot point.

$$I_0 = I_d - md^2$$

i) Compare the obtained values of I_0 estimated by means of these two proposed methods.

Part B (metal ring):

a) Calculate the average time $\bar{t}$ of $n = 100$ oscillations and oscillation period $T = \bar{t}/n$ and its uncertainty $u_c(T)$.

b) Calculate the moment of inertia of the ring I_d and its uncertainty $u_c(I_d)$.

c) Using parallel axis theorem calculate moment of inertia of the ring I_0 around the central axis and its uncertainty $u_c(I_0)$.

d) Using the following equation:

$$I_{0.st} = \frac{1}{8} m(d^2 + D^2)$$

calculate the moment of inertia $I_{o.st}$ around the central axis and its uncertainty $u_c(I_{0.st.})$.

e) Compare the obtained values of the moment of inertia I_o derived from the dynamic and static method.

Part C (metal bar):

a) Calculate uncertainty of the bar lenght $u(l)$ and uncertainties $u(d)$ for each distance between the pivot point and mass center.

b) For each pivot point calculate the average time $\bar{t}$ of $n = 50$ oscillations and its uncertainty $u(\bar{t})$. It can be assumed that the human reaction time is approximately equal to 0.5s.

c) For each pivot point calculate the oscillation period $T = \bar{t}/n$ (n - number of oscillations). Assume $u(n) = 1$. Confirm the dependency of oscillation period of the pendulum on the distance between the pivot point and the mass center.

d) Calculate the moment of inertia I_d of the metal bar and its uncertainty $u_c(I_d)$. Calculations must be performed for each distance d.

e) Confirm the parallel axis theorem. In order to do this verify if for each pivot point the expression $T^2gd - 4\pi^2d^2 = C$ is constant within the margin of estimated uncertainty.

f) Using the obtained value of C for each pivot point calculate moment of inertia around the central axis I_0 and its uncertainty $u_c(I_0)$. Use the following formula:

$$I_0 = \frac{m}{4\pi^2}C.$$

g) Using parallel axis theorem calculate the moment of inertia around the central axis I_0 and its uncertainty $u_c(I_0)$. Repeat calculations for each pivot point.

$$I_0 = I_d - md^2$$

h) Compare the obtained values of I_0 estimated by means of these two proposed methods. Compare the results with theoretical predictions:

$$I_{ot} = \frac{1}{12}ml^2.$$

Variant II (perforated disc, metal bar)

a) Depending on the element chosen follow the instructions included in points a-d or a-c (part A or part C of the 'Result analysis' section)

b) Draw a point plot of T^2d as a function of d^2. Mark error bars on the plot.

c) Use linear regression method to fit the data on the T^2d vs. d^2 plot. Estimate slope a and intercept b of the fitting function $y = ax + b$. Calculate uncertainties $u(a)$ and $u(b)$.

d) Using the estimated value of a calculate $g = \frac{4\pi^2}{a}$ and its uncertainty $u_c(g)$. Check if the obtained value of g is consistent, within the estimated margin of error, with theoretical expectations. The value of $u(a)$ must be determined by means of linear regression method.

e) Using the following formula: $I_0 = \frac{bmg}{4\pi^2}$ calculate the moment of inertia I_o around the mass center and its uncertainty $u_c(I_0)$. The value of $u(b)$ must be determined by means of linear regression method.

6. Suggested tables (to be approved by the supervisor)

Table 1. Perforated disc

Lp.	D_1	D_2	…	t_1	t_2	t_3	…	n	m
	10^{-3} [m]	10^{-3} [m]	10^{-3} [m]	[s]	[s]	[s]	[s]		[kg]
1									
2									
3									
$\bar{X}$									
$u(X)$									

d	$u(d)$	T	$u_c(T)$	I_d	$u_c(I_d)$	I_o	$u_c(I_o)$	C	$u_c(C)$	$C_{śr}$	$u_c(C_{śr})$	I_o	$u_c(I_o)$
10^{-3} [m]	10^{-3} [m]	[s]	[s]	[kgm^2]	[kgm^2]	[kgm^2]	[kgm^2]	[m^2]	[m^2]	[m^2]	[m^2]	[kgm^2]	[kgm^2]

Table 2. Metal ring

m	$u(m)$	n	$u(n)$	d	$u(d)$	D	$u(D)$
10^{-3}[kg]	10^{-3}[kg]			10^{-3}[m]	10^{-3}[m]	10^{-3}[m]	10^{-3}[m]

t_i	$t_{śr}$	$u(t)$	T	$u_c(T)$	I_d	$u_c(I_d)$	I_o	$u_c(I_o)$	$I_{o.st}$	$u_c(I_{o.st})$
[s]	[s]	[s]	[s]	[s]	[kgm^2]	[kgm^2]	[kgm^2]	[kgm^2]	[kgm^2]	[kgm^2]

Table 3. Metal bar

m = *u(m)* = *l* = *u(l)* =

d_1	$u(d_1)$	t_i	t_{sr}	$u(t)$	T	$u_c(T)$	I_d	$u_c(I_d)$	I_o	$u_c(I_o)$	$I_{o.st}$	$u_c(I_{o.st})$
10^{-3} [m]	10^{-3} [m]	[s]	[s]	[s]	[s]	[s]	[kgm^2]	[kgm^2]	[kgm^2]	[kgm^2]	[kgm^2]	[kgm^2]

d_1	$u(d_1)$	C	$u_c(C)$	$C_{śr}$	$u_c(C_{śr})$	I_o	$u_c(I_o)$
10^{-3}[m]	10^{-3}[m]	[m^2]	[m^2]	[m^2]	[m^2]	[kgm^2]	[kgm^2]

Variant II

Pivot point number	d_i	$u(d_i)$	T_i	$u(T_i)$	d_i^2	$u(d_i^2)$	T_i^2	$u(T_i^2)$	I_0	$u_c(I_o)$
	10^{-3} [m]	10^{-3} [m]	[s]	[s]	10^{-3} [m^2]	10^{-3} [m^2]	[s^2]	[s^2]	[kgm^2]	[kgm^2]
1										
2										
3										