



**Exercise  
28**

**DETERMINATION OF THE THERMAL CONDUCTIVITY OF  
INSULATORS**

**Aim of the exercise:** determination of the thermal conductivity of an insulator.

**Issues:** heat transfer, thermal conduction, thermal radiation, convection, the thermal conductivity, measurement method of the thermal conductivity of insulators.

**1. Theoretical background**

**1.1. Introduction**

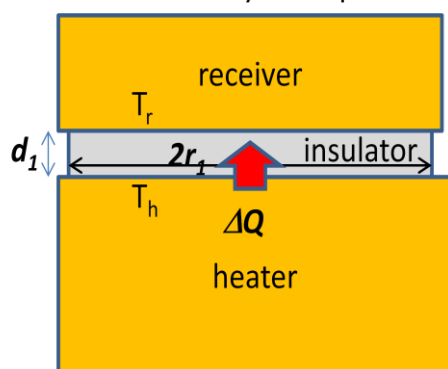
The phenomenon of energy transfer between parts of a system caused by the different temperatures of the parts is called heat transfer. There are three mechanisms of heat transfer in nature: conduction, radiation and convection.

**1.2. Thermal conduction**

The thermal conduction plays a significant role in solids. Heat (i.e. the energy associated with thermal motions of atoms and electrons) always flows freely from a warmer to a cooler place. The opposite direction is also possible but only when it is forced (like in a refrigerator) by the additional energy introduced to the system.

In an insulated system the heat is transferred freely until the thermal equilibrium is reached, i.e. until the temperatures of all parts of the system are the same.

If a certain body is kept between two different temperatures (like the insulator in the fig.1) the rate of the heat transfer is given by the following equation:



$$\frac{\Delta Q}{\Delta t} = k \cdot S_1 \cdot \frac{T_h - T_r}{d_1} \quad (1)$$

**Fig.1.** An insulator kept between two different temperatures (i.e. between the temperature of the heater  $T_h$  and the temperature of the receiver  $T_r$ ). In our setup the the insulator, heater and receiver have the cylindrical shape.

where:

$\Delta Q/\Delta t$  – the rate of thermal conduction (the amount of heat transferred per a unit of time),

$k$  – the thermal conductivity,

$S_1$  – the area of the insulator perpendicular to the direction of thermal conduction (in our case  $S_1 = \pi r_1^2$  where  $r_1$  is the radius of the slab of the insulator),

$d_1$  – thickness of the insulator,

$T_h$  and  $T_r$  are the temperatures on both sides of the insulator (i.e. the temperature of the heater and receiver respectively).

Good heat conductors are materials through which heat can be easily transferred and have high thermal conductivities  $k$ . Materials with low thermal conductivities ( $10^{-1} - 10^{-2} \text{ J}/(\text{s} \cdot \text{m} \cdot \text{K})$ ) are called insulators.

From a microscopic point of view, heat conduction in solids is carried out by oscillations of the crystal lattice and by the so called free electrons from conduction band which are not bound to particular atoms so can move in the whole crystal lattice. That is why the thermal conductivity can be written as  $k = k_c + k_e$ , where  $k_c$  represents the contribution of the crystal lattice and  $k_e$  of the free electrons. If the free electrons belonging to the conduction band exist, like in metals,  $k_e$  may be two orders of magnitude bigger than  $k_c$ . This is the reason for which thermal conduction of metals is much better than of dielectrics in which free electrons do not exist.

### 1.3. Convection

Convection takes place in fluids i.e. in liquids and gases. If a liquid enclosed in a vessel is heated from below, the lower part of the liquid expands and its density decreases. Due to the buoyant forces, this heated portion of the liquid goes up and with it the heat is transferred up too. Convection is when heat is transferred along with the fluid. Of course the heat conduction also take place in fluids but is much less efficient than convection so can be neglected in most cases. The traveling masses of fluid in the process of convection are called currents. The currents can be divided into natural and forced currents. The natural currents can be observed in the atmosphere (winds) or in oceans. The forced convection occurs e.g. while stirring or when you use a fan on a hot day.

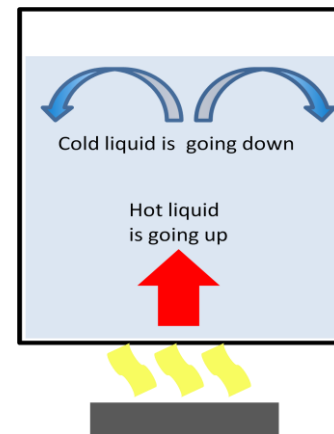


Fig. 2. Convection

### 1.4. Radiation

In this mechanism a body exchanges heat with its surroundings by the electromagnetic waves which in this case are called thermal radiation. The body emits the thermal radiation at the expense of its internal thermal energy whereas the body absorbing thermal radiation increases its thermal energy. Unlike conduction and convection, the heat transfer by thermal radiation does not require any medium. Moreover, heat transfer by thermal radiation is the most efficient in a vacuum. The rate  $P_e$  at which an object emits energy by the thermal radiation (i.e. the emitted energy per a unit of time) is proportional to its temperature (in kelvins) to the power of 4:



Fig. 3. Radiation

$$P_e = \alpha(T)\sigma S T^4 \quad (2)$$

where:  $\sigma$  is called the Stefan-Boltzmann constant,  $\alpha(T)$  represents the emissivity of the object's surface which belongs to the range from 0 to 1 (1 is for a black body radiator which in fact does not exist in nature, but a model of a black body with  $\alpha(T)$  close to 1 can be easily done),  $S$  is the area of the emitter's surface. So every object emits the thermal radiation because every body in nature has the temperature higher than 0 K even those extremely cold whose temperature may be very close to zero kelvins but never exactly zero.

Bodies emit thermal radiation to the environment and at the same time absorb thermal radiation coming from the surrounding environment. The rate  $P_a$ , at which an object absorbs energy by thermal radiation coming from the environment which is at the temperature  $T_e$  can be expressed by formula:

$$P_a = \alpha(T)\sigma S T_e^4 \quad (3)$$

Due to the fact that for a black body  $\alpha(T) = 1$  the black body absorbs all the incident electromagnetic radiation which means that no incident radiation is reflected from its surface. But we should also remember that a black body at a certain temperature emits its own radiation given by the equation 2 with  $\alpha(T) = 1$ . So at a certain temperature a black body emits more radiation per unit of its area than other bodies for which  $\alpha(T) < 1$ . It seems to be a paradox but it's not. First of all we should distinguish between reflected and emitted light. Imagine that we look at two bodies at the same room temperature in daylight, and one body is white and the other is the model of a black body. Which of these bodies emits more thermal radiation per unit of area? Of course, it is the black one which is closer to an idealized black body with  $\alpha(T) = 1$  (eq. 2). So why is the white body brighter? Because it reflects much more daylight which adds to its emitted light. The idealized black body does not reflect any light whereas the idealized white body reflects all the incident light.

If we eliminate the external light (emitted or reflected by the surroundings) and keep the two bodies in the complete darkness but still at room temperature, we would not see both bodies with the naked eye, despite their thermal radiation. This would be because at room temperature thermal radiation occurs mainly in the infrared (not-visible) range. To convince ourselves that the black body's thermal emission is stronger, at the room temperature we would have to observe the bodies with night vision goggles. Or we would have to heat the two bodies up to, say, 1000 K. At such a high temperature the thermal radiation is shifted towards the visible range (according to Wien's displacement law) so we would be able to notice that the black body is brighter even without the goggles.

Knowing that at the same time an object emits its own light to the environment and absorbs the light from the environment, we can express the formula for the net rate of the thermal energy which the object exchanges with the surroundings due to the radiation:

$$P = \frac{\Delta Q}{\Delta t} = P_{ab} - P_{em} = \alpha(T)\sigma S(T_e^4 - T^4) \quad (4)$$

Positive  $P$  means that due to the thermal radiation the object increases its internal energy.

## 2. The measurement setup and experimental procedure

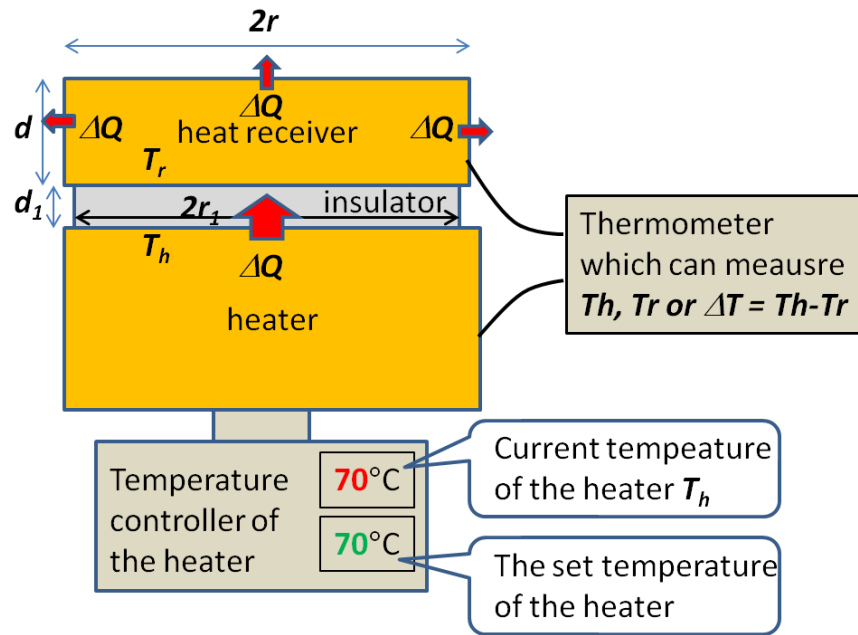


Fig. 4. Scheme of the measuring system in part 1. The insulator, the heater and the receiver have the cylindrical shape.

The components of the measurement setup:

**heater** - the source of heat at the temperature  $T_h$ ,

**heat receiver** - of the thickness  $d$  and diameter  $2r$  at the temperature  $T_r$ ,

**insulator** - of thickness  $d_1$  and diameter  $2r_1$ ,

**Temperature controller of the heater**,

**Thermometer** which can measure in the version 1:  $T_h$ ,  $T_r$  or  $\Delta T = T_h - T_r$  or in the version 2: only  $\Delta T = T_h - T_r$ .

### Part 1

Our task is to calculate the thermal conductivity  $k$  from the eq. 1. To achieve this aim first we need to estimate the rate  $\Delta Q/\Delta t$  at which the heat is transferred through the insulator in the setup represented in fig. 4. To do this the exercise is divided into two parts. In the first part we build the setup presented in the fig.4. Then we set the temperature of the heater to a certain value (usually  $T_h = 70^\circ\text{C}$ ). After that we wait until the thermally steady state is reached in the system i.e. the measured  $\Delta T$  becomes stable (it is a kind of equilibrium). We record the value as  $\Delta T_{eq}$ .

Using eq. 1 and knowing that the area of the insulator perpendicular to the direction of the heat conduction is  $S_1 = \pi r_1^2$  and that in the equilibrium the difference  $\Delta T_{eq} = T_h - T_r$  was measured directly we can write:

$$\frac{\Delta Q}{\Delta t} = k \cdot \pi r_1^2 \cdot \frac{\Delta T_{eq}}{d_1} \quad (5)$$

where:

$k$  - the thermal conductivity,

$\Delta Q/\Delta t$  - the rate of heat conduction through the insulator in the setup represented in fig. 4.

$\Delta T_{eq}$  - the difference of the heater and receiver temperatures in the equilibrium.

## Part 2

In the thermal equilibrium in part 1 the rate of heat conduction through the insulator  $\Delta Q/\Delta t$  is equal to the rate of the heat dissipation by the heat receiver i.e. by its upper and side surfaces (red arrows in fig. 4). To evaluate the rate we need to take the heated receiver out of the insulator and to determine from measurements the rate of the heat dissipation  $\Delta Q_1/\Delta t$  by the free-standing receiver (fig. 5) during its cooling down.

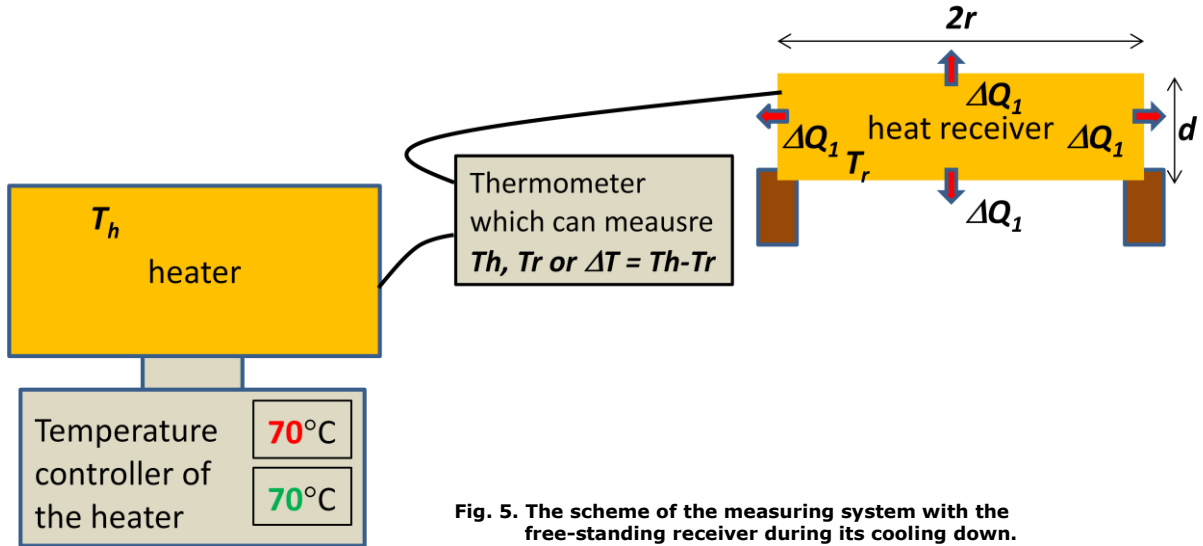


Fig. 5. The scheme of the measuring system with the free-standing receiver during its cooling down.

From the general physics we know that the relation of the heat dissipated from the free-standing receiver and the change in its temperature is as follows:

$$\Delta Q_1 = m \cdot c \cdot \Delta T_r \quad (6)$$

where:  $m$  - mass of the receiver,

$c$  - the specific heat of the material the receiver was made of.

$\Delta T_r$  - the change in temperature of the heat receiver during its cooling down.

So:

$$\frac{\Delta Q_1}{\Delta t} = m \cdot c \cdot \frac{\Delta T_r}{\Delta t} \quad (7)$$

where  $\Delta T_r/\Delta t$  is the rate at which the temperature of the receiver decreases. In our case we measure  $\Delta T$  i.e. the difference between the constant temperature of the heater (which still is usually kept about  $T_h = 70^\circ\text{C}$ ) and the temperature of the free-standing receiver  $T_r$ . Due to the fact that the temperature of the heater is kept constant the change  $\Delta T_r$  is the same as the measured  $\Delta T$ . So  $\Delta T_r/\Delta t = \Delta T/\Delta t$ .

Therefore eq. 7 can be written in the form:

$$\frac{\Delta Q_1}{\Delta t} = m \cdot c \cdot \frac{\Delta T}{\Delta t} = m \cdot c \cdot n \quad (8)$$

To get the cooling rate  $\Delta T/\Delta t$  (denoted by  $n$ ) we measure the  $\Delta T$  every **10** seconds. Then the graph  $\Delta T$  vs. time  $t$  (i.e.  $\Delta T = f(t)$ ) should be plotted. From the graph the rate  $n$  should be found out as the slope of the linear part of the plotted line in the vicinity of the thermal equilibrium achieved in part 1 (i.e. close to  $\Delta T_{eq}$ ).

In the fig.4  $\Delta Q$  means the heat dissipated by the heat receiver which is put on the insulator whereas in the fig.5  $\Delta Q_1$  means the heat dissipated by free-standing receiver. So at the same temperature  $\Delta Q_1 > \Delta Q$  because the  $\Delta Q_1$  is dissipated by the whole

surface of the cylindrical receiver (two bases and the side of the cylinder) while  $\Delta Q$  is dissipated only by the upper base and the side of the receiver. Assuming that the dissipated heat is directly proportional to the area of the surface through which the heat is dissipated (which is quite a good simplification) we can get the needed  $\Delta Q$ :

$$\Delta Q = \frac{S_b + S_s}{2S_b + S_s} \Delta Q_1. \quad \text{So:} \quad \frac{\Delta Q}{\Delta t} = \frac{S_b + S_s}{2S_b + S_s} \frac{\Delta Q_1}{\Delta t} \quad (9) \text{ i } (10)$$

where:

$S_b = \pi r^2$  - area of the base of the cylindrical receiver,

$S_s = d \cdot 2\pi r$  - area of the side of the receiver.

Finally from the equations (5), (8) and (10) we can derive the formula for the thermal conductivity:

$$k = \frac{m \cdot c \cdot n \cdot d_1 \cdot (r + 2d)}{2\pi \cdot r_1^2 \cdot \Delta T_{eq} (r + d)} \quad (11)$$

where  $\Delta T_{eq}$  is the difference between the temperature of the heater and the temperature of the receiver measured in part 1 of the exercise when the system was in the thermal equilibrium.

### 3. Do the analysis of the uncertainty of the calculated thermal conductivity.

#### 4. Questions:

1. Name and explain different mechanisms of heat transfers.
2. Explain the mechanisms of thermal conductivity in metals and dielectrics.
3. Explain what convection is.
4. On which parameters does the rate of thermal radiation depend on?
5. Assuming that a black body and, say, a yellow body are at the same temperature which of them emits more energy by thermal radiation (in the same time and from the same area). Justify your answer.
6. Describe the method of determining the thermal conductivity used in this exercise.

opracował: Janusz Bożym