EXERCISE 29	DETERMINATION OF THERMAL EXPANSION COEFFICIENT AND HEAT TRANSFER INVESTIGATION

Exercise Objective: Measurement the elongation of the wire as a function of temperature and determining the linear coefficient of thermal expansion; familiarization with heat exchange processes and determination the effective heat transfer coefficient.

Topics: interatomic interactions in a solid, thermal expansion phenomenon, thermal expansion coefficient, heat transfer mechanisms.

1. Introduction

Thermal expansion phenomenon

The thermal expansion phenomenon, in macroscopic terms, is the change in the size of bodies caused by a change in temperature. Increased body size corresponds at the microscopic level to a greater average distance between its atoms. An increase in temperature causes an increase in average interatomic distances in a body.

The phenomenon of thermal expansion can be either desirable or undesirable. For example, an undesirable thermal expansion effect occurs when railroad rails deform/bend on a hot day. Another example is the shrinkage of power lines or telephone wires in winter, which can cause them to break, so they are hung loosely and characteristic arches are created to prevent them from breaking in winter. The thermal expansion phenomenon is used in metrology and engineering, and devices such as bimetallic thermostats, irons, kettles, and liquid thermostats in cars.

To explain the phenomenon of thermal expansion in the case of solids we can use the model of the structure of a solid. Solids have a crystalline or amorphous (non-crystalline) structure. Crystals consist of atoms or molecules arranged in the space of a solid in an orderly manner, forming the so-called crystal lattice (Fig. 1). The mutual distances of atoms are about 10^{-10} m.

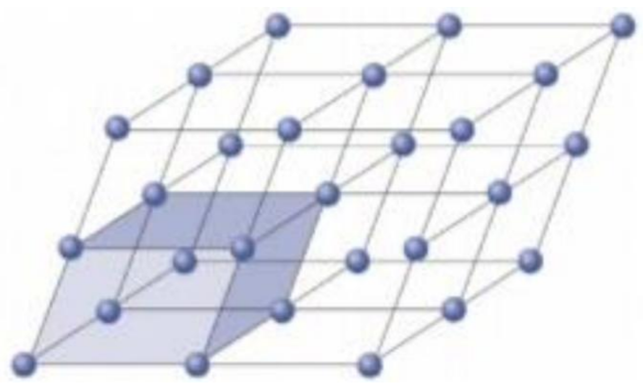

Fig. 1. A model of a simple crystal lattice, showing a crystal consisting of so-called elementary cells (one, one, for example, marked in blue). In the corners of each elementary cell are atoms (blue balls).

Atoms in a crystal lattice oscillate around their equilibrium positions. The amplitudes of these oscillations as well as the average interatomic distances as a rule to increase as the temperature increases, causing the solid to expand. The increase in average interatomic distances should be related to the interactions between the atoms of the solid. Figure 2 shows a graph of the interaction forces between two atoms depending on their mutual distance $\boldsymbol{r}$.

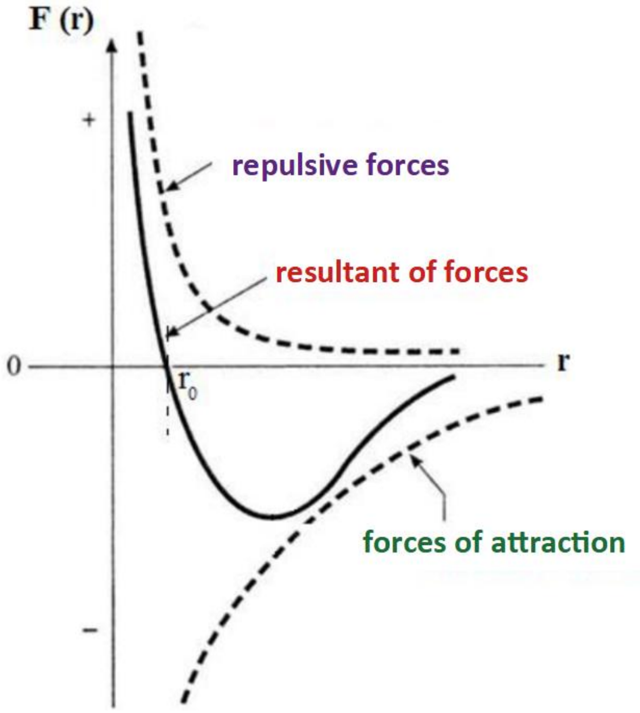

Fig. 2. Graph of the dependence of interatomic interaction forces F(r) as a function of the distance between two atoms ***r***. The symbol $\boldsymbol{r_o}$ denotes the equilibrium position of the atom.

There are both attractive and repulsive forces between atoms of solids. From the figure above, it can be seen that for $\boldsymbol{r} = \boldsymbol{r_o}$ the forces of repulsion and attraction balance each other and the resultant force is zero. In this case, the atom does not change its equilibrium position. In contrast, atoms are said to swing out of their equilibrium position when $\boldsymbol{r} > \boldsymbol{r_o}$ - the forces of attraction then prevail (***F(r)*** < 0, i.e. the turn of the force is opposite to the turn of the ***r*** axis), and for $\boldsymbol{r} < \boldsymbol{r_o}$ - the forces of repulsion prevail (***F(r)*** > 0).

Based on the relationship ***F(r)***, the dependence of the potential energy as a function of the distance between two atoms $\boldsymbol{E_p(r)}$ can be determined. The graph of ***Ep(r)*** is shown in Fig. 3.

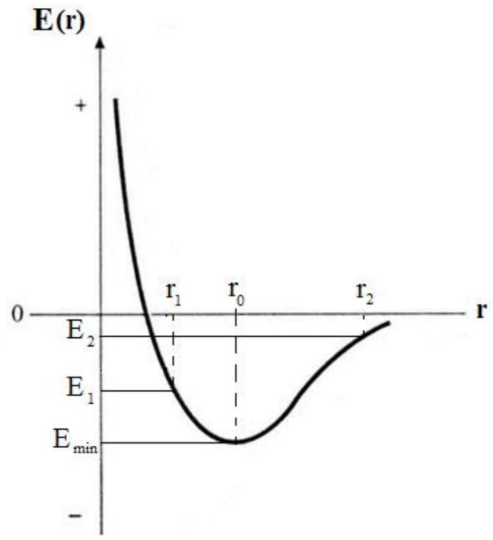

Fig. 3. Graph of potential energy as a function of the distance between two atoms.

An atom with total energy ***E*** oscillates around the point $\boldsymbol{r_o}$, which corresponds to the minimum of potential energy ($\boldsymbol{E_{min}}$). Oscillations of atoms cause changes in their mutual distances in the interval from r_1 to r_2. Thus, atoms move in the interval from r_1 to r_2. As can be seen in Figure 3, due to the asymmetry of the $E_p(r)$ curve, at higher temperature the center of oscillation shifts to the right relative to the equilibrium position r_o, which corresponds to an increase in the average distance between neighboring atoms. The curves in Figures 2 and 3 represent the forces and energies of the interactions of only two atoms, although it should be borne in mind that each atom interacts with multiple atoms simultaneously. The temperature-induced increase in the distance of neighboring atoms is small, but multiplied by the number of atoms gives an experimentally observable change in the dimension of the body.

The dependence of ***F(r)*** near the equilibrium position r_o is described by the approximate relation:

$$F = -cx+bx^2, \qquad (1)$$

where c and b are constants. The member $F = -cx$, appearing in equation (1), refers to harmonic oscillations (Hooke's law), while $F = bx^2$ is a nonlinear member, describing deviations from the harmonic character of oscillations. Taking the dependence of the strength of interatomic interactions on the mutual distance between atoms, described by equation (1), it can be shown that, as a result of thermal oscillations, the average distance between atoms of a solid body differs from r_o by a value x, proportional to the absolute temperature of the body:

$$(2) \qquad \bar{x} = \frac{bkT}{c} \,,$$

where: k - Boltzmann's constant, T - absolute temperature.
The increase the average distance between atoms of a body during its heating is the reason for linear and volumetric expansion of the body. For quantitatively describe the phenomenon of linear thermal expansion, the linear expansion coefficient is introduced, defined by the following equation:

$$(3) \qquad \alpha = \frac{1}{r_0}\frac{d\bar{x}}{dT} \,.$$

The linear coefficient of thermal expansion shows by what fraction the object's length increases when the temperature increases by one degree. Substituting dx/dT, calculated from equation (2), into equation (3), we obtain:

$$\alpha = \frac{bk}{r_0 c} \,. \qquad (4)$$

In this way, the values of coefficients α for different materials can be calculated. Table 1 shows examples of values of linear expansion coefficients α for several selected materials.

Table 1. Values of the linear expansion coefficient of selected substances.

Substance	α **(10^{-6} / °C)**	**Substance**	α **(10^{-6} / °C)**
Ice (0°C)	51	Steel	11
Lead	29	Glass	9
Aluminum	23	Diamond	1.2
Brass	19	Quartz	0.5
Copper	17	Concrete	12

The next task concerns heat transfer processes. It should be noted that heat transfer processes are very important for many engineering fields, e.g. construction, mechanics, physics or biomedical engineering. In addition, there are many devices (such as a car radiator, water heater) whose main function is the exchange of heat between two (or more) fluids. This general term refers to a wide group of devices that differ in purpose, construction and method of implementing heat flow, so it is necessary to systematize knowledge of the above-mentioned phenomena.

1.2 Heat exchange processes – basic concepts

Heat exchange is a concept that covers the entire range of issues related to the transfer of heat between bodies or parts of one body, caused by a temperature difference. The direction of the phenomenon is determined by the second law of thermodynamics. According to it, energy in the form of heat is transferred from an area with a higher temperature to an area with a lower temperature, and this exchange is subject to the law of energy conservation.

The main goal in solving heat transfer problems is the calculation of the heat flow dQ/dt [J/s=W] transferred in the system under consideration. We distinguish three main mechanisms of heat transfer: convection, radiation and conduction.

Convection is a method of heat transfer by means of "flows" occurring in liquids and gases, and is achieved by mixing flows or particles with different temperatures. This process is caused by local differences in the density of the liquid. The main law describing convection is Newton's law:

$$\frac{dQ}{dt} = h_{eff} \cdot S \cdot \left(T - T_0\right) \; , \qquad (5)$$

where: h_{eff} – effective heat transfer coefficient [W/(m^2*K)], which is a function of many parameters, such as: jet velocity, liquid density, viscosity, T – surface temperature of the test body, T_0 – ambient temperature (initial).

Heat transfer by radiation consists in the emission of electromagnetic waves by a body with a temperature above absolute zero. During this process, the body's internal energy is converted into electromagnetic radiation. This occurs because the thermal motion of charged particles (such as electrons) causes them to accelerate and oscillate, which leads to the emission of electromagnetic waves. When this radiation encounters other bodies (or parts of the same body), it is partially or completely absorbed, being converted back into internal energy. If the amount of energy radiated by the surface is different from the amount absorbed, heat exchange occurs. According to the Stefan-Boltzmann law, the energy flux emitted by the body increases proportionally to the fourth power of its absolute temperature. In this case, the equation describing heat transfer by radiation is as follows:

$$\frac{dQ}{dt} = \varepsilon \cdot \sigma_c \cdot S \cdot \left(T^4 - T_0^4\right) \; , \qquad (6)$$

where: ε is the radiation coefficient, σ_c is the Stefan-Boltzmann constant equal to 5.67×10^{-8} [W*m^{-2}*K^{-4}].

Heat conduction is the exchange of heat between directly contacting parts of the same or different bodies, consisting in the transfer of kinetic energy by molecules making microscopic motions. The main cause of heat conduction of a body is the temperature difference between its parts. This type of heat transfer takes place in gases, liquids and solids. The expression describing this effect is as follows:

$$\frac{dQ}{dt} = \lambda \cdot S \cdot \frac{T - T_0}{l} \; , \qquad (7)$$

where: λ – thermal conductivity coefficient [W/(m*K)], l – length of the tested body.

2. Measurement principle and measuring setup

By studying the phenomenon of thermal expansion of solids, we determine the dependence of the change in its length on temperature. From formula (8) we obtain the dependence of the length of the body l on the temperature T:

$$l_t = l_0(1+\alpha\Delta T) \ , \qquad (8)$$

where: l_t – body length at temperature T, l_0 – body length at temperature T_0, $\Delta T = T - T_0$, α – linear expansion coefficient. The above equation is true in a certain temperature range. Transforming equation (8) to the following form:

$$\frac{l_t - l_0}{l_0} = \alpha\Delta T \ , \qquad (9)$$

we obtain an expression from which it follows that, based on the measured dependence of the elongation of the body on the temperature increment (ΔT), we can determine the coefficient of linear expansion α by first calculating the relative elongation of the body $\left(\frac{l_t - l_0}{l_0}\right)$. The value of the coefficient α corresponds to the directional coefficient of the slope of the straight line on the graph showing the dependence of the body's relative elongation $\Delta l/l_0$ on the temperature increase ΔT.

The measurement method consists of heating the sample and measuring its elongation. The length increase is caused by the body temperature rise. A schematic of the measuring setup for the linear expansion study is shown in Figure 4.

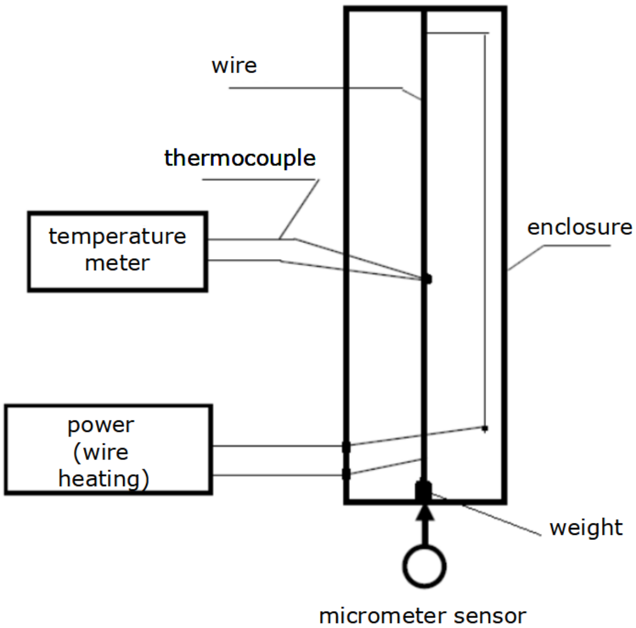

Fig. 4. Measuring system scheme.

In this exercise, the sample is in the form of a wire (Fig. 4). The temperature of the wire is increased by heating it with an electric current flowing directly through it. Current values are indicated for a DC source. In turn, the voltage values applied to the wire are read from a voltmeter. The temperature of the wire is measured using a thermocouple connected to a digital temperature converter. The temperature of the wire must be constant along its entire length, so to minimize the adverse effects of air, the wire under test is mounted in a housing with a transparent window. Each change in current increases the wire's temperature. Temperature readings should be taken after its stabilization, i.e., when the amount of Joule heat released in the wire per unit of time is equal to the energy transferred to the environment. The wire is loaded with a weight that maintains a constant slight tension on it. As the wire heats up, its length increases, and the weight goes down. The elongation is measured using a micrometer sensor (see materials in the "Study Aids" tab – "Principles of Using Measuring Instruments").

3. Tasks to complete

A) Measurements: Set up the measuring system according to the scheme shown in Fig. 4. Turn on the temperature meter and read the initial temperature. Measure the wire diameter. Gradually increase the current in the circuit (by 0.2 A up to 80 °C, by 0.1 A above 80 °C until reaching a temperature of about 120 °C). For a given current intensity value, note the supply voltage values and the temperature after it has stabilized. During measurements, read the values of the increase in wire length as a function of temperature in the range from room temperature to approx. 120^{o}C. Open the wall of the measuring chamber halfway up and repeat all measurements.

B) Processing the results: Plot a graph of the dependence of the body's relative elongation on the temperature increase $\Delta l/l_0=f(\Delta T)$ for a closed and open measuring chamber. Plot the so-called uncertainty rectangles on the graph. Use the linear regression method to determine the coefficient α of linear thermal expansion of the tested wire. Estimate its uncertainty. Compare the values of α in both cases. Knowing the value of α, evaluate what material the wire tested in the experiment was made. Plot a graph of the dependence of power on the temperature difference $P=f(\Delta T)$ for both cases – i.e. a closed and open measuring chamber. Plot the uncertainties of the power measurements at the beginning, middle and end of the graph. Explain why the graph of the dependence of power on temperature is non-linear, and the power needed to maintain the set temperature is greater in the case of an open measuring chamber.

4 Questions:

(1) Graphically present and discuss interatomic interactions in a solid (force, energy).
(2) What is the phenomenon of thermal expansion in macroscopic and microscopic terms?
(3) Define the thermal coefficient of linear expansion (unit). Is this coefficient a constant for a given body?
(4) Discuss the method of determining the thermal coefficient of linear expansion.
(5) Characterize the mechanisms of heat exchange.
(6) Explain why the power needed to obtain a given temperature is greater when the measuring chamber is open.

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