



**Exercise
44**

**THE MEASUREMENT OF THE DEPENDENCE OF RESISTANCE
OF METALS AND SEMICONDUCTORS ON TEMPERATURE**

Goal: The measurement of resistance of metal and semiconductor as a function of temperature and calculation of the temperature coefficient of resistance of metal and energy gap of semiconductor.

Key words: resistivity of metals and semiconductors, concentration and mobility of free carriers of charge, intrinsic and doped semiconductors, band structure, energy gap.

1. Introduction

In terms of electrical properties solid states can be divided into three groups: conductors, semiconductors and insulators. What makes them different is the concentration of so called free electrons which are the carriers of charge. Free electrons are not bound to the particular atoms, but can move across the crystal lattice. We say about these electrons that they occupy states in a conduction band.

Metals

In metals concentration of free electrons n , i.e. number of electrons per volume unit, is very high (e. g. in cuprum $n = 8,5 \cdot 10^{28} \text{ m}^{-3}$) and is of the same order of magnitude as concentration of atoms creating the crystal lattice from which the electrons come. The concentration of free electrons practically does not depend on temperature. So another quantity called mobility of electrons affects the resistivity.

Mobility is connected with the average velocity of all electrons called the drift velocity \vec{v}_d caused by the electric field which appears thanks to the voltage applied to the sample. This velocity should not be mistaken for the velocities of the chaotic thermal movements of particular electrons which at the room temperature are of an order of 10^6 m/s . In spite of the thermal movements without electric field the drift velocity is equal to zero because in average the same number of electrons move in a certain direction as in the opposite direction. Only when we apply electric field the whole gas of free electrons starts to move along the electric field with the drift velocity which is relatively small (of an order of 10^{-4} m/s). However the value of the velocity is not a good parameter which describes the features of the electrons in the material because its value strongly depends on the external electric field. So to obtain a parameter which describes the ability of electrons to move in the material because of the external electric field but which would not be dependent on this field a new parameter was defined which is called mobility μ . It is defined as a ratio of the drift velocity and the applied electric field:

$$\mu = \frac{\vec{v}_{dr,u}}{E}. \quad (1)$$

In metals in a certain electric field when the temperature increases the drift velocity of the free electron gas decreases (although at the same time the velocities of particular electrons attributed to thermal movements increase). Why the drift velocity decreases? The reason is simple. At higher temperature the atoms in crystal lattice oscillate with bigger amplitudes so the effect of scattering electrons by the atoms is more significant. This process causes the electrons to move with smaller drift velocity so their mobility decreases.

Summarizing, in metals the concentration of free electrons is very high and practically does not change with increasing temperature but mobility decreases. That is why the electric conductivity which can be express by the equation:

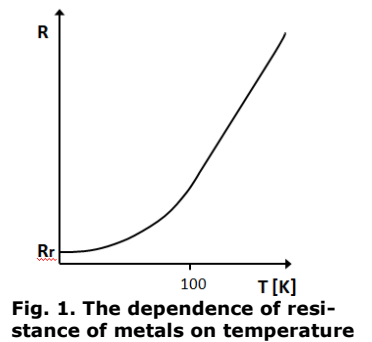
$$\sigma = en\mu \tag{2}$$

decreases too. Where: e – elementary charge, n – concentration of free electrons, μ – mobility of free electrons.

Electric resistivity is the reciprocal of conductivity so the resistivity increases when the temperature goes up. Above a certain temperature (about 100 K, but different for different metals) the resistance of a metallic sample depends linearly on temperature (fig. 1.) and can be expressed by the equation:

$$R_{m,t} = R_{m,o}(1 + \alpha \cdot \Delta T) \tag{3}$$

where: $R_{m,o}$ – resistance of the metal at the initial temperature T_o ,
 $R_{m,T}$ – resistance of the metal at the temperature $T = T_o + \Delta T$.
 α – temperature coefficient of resistance (different for different metals). The bigger is the value the more sensitive is the resistance of the metal to the change of temperature.



Below a certain temperature the oscillations of atoms almost vanish and the resistance of the metal depends mainly on the defects of the crystal lattice (i.e. any deformity which destroys the periodicity of the crystal lattice like dislocations, vacancies, impurities and so on). As can be noticed in the fig.1. in this range of low temperatures the resistance of the metallic sample increases when temperature increases but not linearly. At the temperature near the absolute zero remains only the resistance connected with the imperfection of the crystal lattice so its value gives us the information about purity and regularity of crystal lattice of the investigated sample.

Semiconductors

A. Intrinsic semiconductors

Intrinsic semiconductors are pure semiconductors which do not contain any dopants (called also impurities). In semiconductors the concentration of free electrons strongly depends on temperature. At the temperature near 0 K this concentration is practically equal to zero. At so

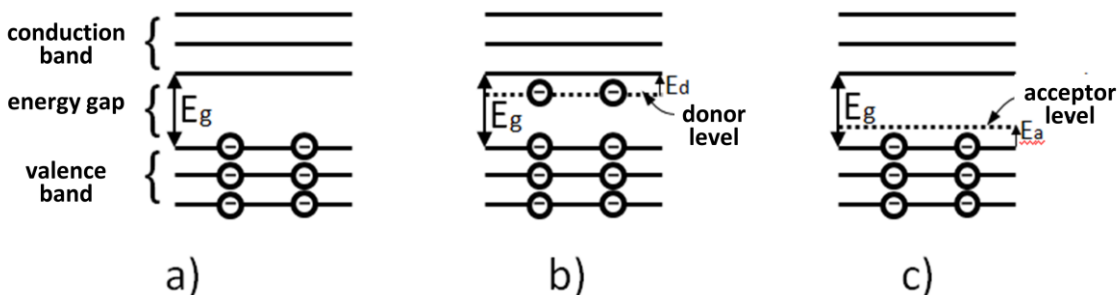


Fig. 2. Band structure of semiconductor. E_g - energy gap.
a) Intrinsic semiconductor at temperature $T \approx 0K$.
b) n type semiconductor (with donors). E_d - activation energy of donors.
c) p-type semiconductor (with acceptors). E_a - activation energy of acceptors.

extremely low temperatures atoms creating the crystal lattice do not deprive themselves of electrons, even the weakly bound with them which are called valence electrons. In such a case we say that valence electrons entirely fill up the valence band which is also called fundamental band (fig. 2a). These bound electrons cannot conduct the electric current. In this situation the

conduction band is empty which means that there are not free electrons in the sample, which is why the resistivity of the sample is huge. The conduction band is separated from the valence band by, so-called, energy gap whose width for semiconductors usually does not exceed 3 eV. It means that to obtain free electrons in the conduction band we need to supply appropriately high energy to the electrons occupying the valence band in order to enable them to jump through the energy gap. In other words the supplied energy enables the electrons to overcome the binding energy with atoms so they became "free" which means that they can move in the whole crystal lattice and conduct the current. One of the ways of supplying the needed energy is heating the sample. When the temperature of the semiconductor is sufficiently high the concentration of free electrons increases exponentially with increasing temperature. This is very strong increase so in spite of the fact that the mobility of free electrons in this range of temperature, like in metals, decreases with increasing temperature the conductivity (eq. 2) of semiconductor increases like concentration of free electrons i.e. exponentially. So the resistance of the semiconductor sample decreases exponentially with increasing temperature, what can be expressed by the equation:

$$R_s = R_{os} \exp\left(\frac{E_g}{2kT}\right) \quad (4)$$

where: R_{os} — approximately constant value which depends on the kind of the semiconductor and its dimensions,
 E_g — energy gap dependent on the kind of semiconductor,
 T — temperature expressed in Kelvins,
 k — Boltzmann constant.

From the equation (4) it can be concluded that at a certain temperature the wider is the energy gap of the semiconductor the bigger is its resistance.

It is worth noticing that even at high temperatures the concentration of free electrons in semiconductors is still a few orders of magnitude lower than in metals, so semiconductors have bigger resistivity.

One of the examples of a being commonly used intrinsic semiconductor is silicon (Si) which creates a regular crystal lattice. Silicon belongs to the IV group in the periodic table of elements. It means that it has four valence electrons which create bonds with four neighbouring atoms (fig. 3). In this situation each atom is surrounded by its own four valence electrons and by additional four valence electrons (one electron from one neighbouring atom). Such a state with eight valence electrons around each atom of silicon is in accordance with octet rule and is the most preferable from the minimal energy point of view so such crystal lattice is stable.

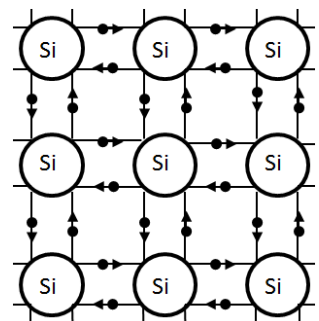


Fig. 3. Intrinsic semiconductor (without dopants) at the temperature $T \approx 0K$ (all electrons are bound to atoms)

When a valence electron e (fig. 4) breaks free (e.g. due to heating the sample) it leaves the atom so the atom becomes positively ionized (Si^+). Such positively ionized atom in crystal lattice is called a "hole". One atom may give only one free electron because the other valence electrons are bound to atoms much stronger. Knowing how free electrons and holes come into being it is obvious that in the intrinsic semiconductor the concentrations of electrons and holes are equal.

The free electrons and holes can conduct an electric current but the holes can conduct the current in a different way. When we apply voltage to the sample the electric field caused by the voltage together with the electric field coming from the ion (Si^+) attract one valence electron from the neighbouring neutral atom (Si). So the neutral atom from which valence electron was taken becomes positively ionized but the previously ionized atom to which the valence electron was attracted becomes neutral. This process is presented in fig. 4a (before changing position) by

dashed line denoted by 1. So it seems as if the ion moves in the $V+$ direction (dashed line 1' in fig. 4a) but in fact the valence electron moves in the opposite direction. The whole process repeats many times (lines 2 and 2' in fig. 4b) until the ion i.e. the hole reaches the negative potential of the applied voltage $-V$. Of course the total electric current results from the current of both free electrons (like electron e in the fig. 4 moving with the velocity \vec{v}_e to the $+V$ potential) and holes (like Si^+ moving to $-V$ potential). Taking the free electrons and holes into consideration the general formula for resistance of intrinsic semiconductor is still expressed by the equation (4).

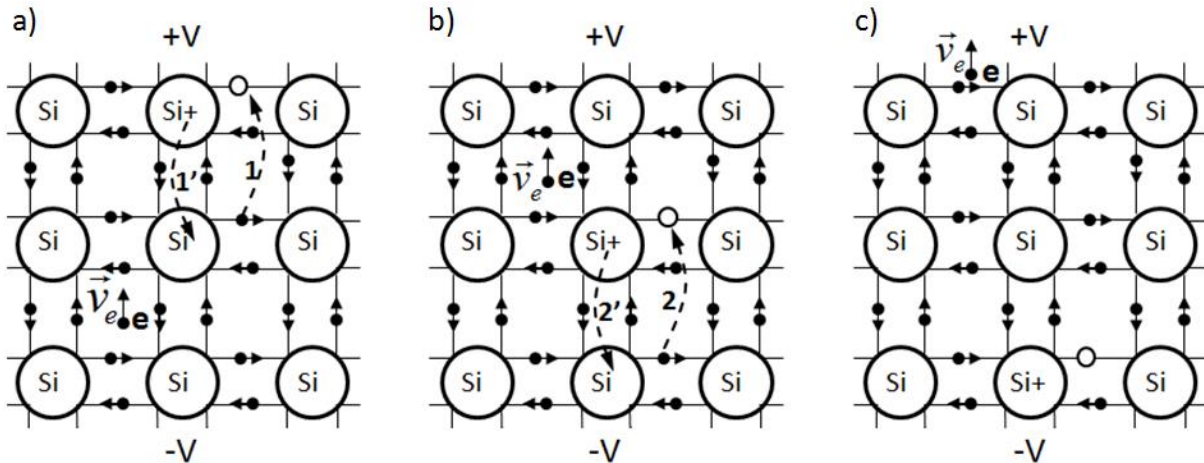


Fig. 4. The figure explaining electric conduction caused by free electrons and holes. The free electrons conduction is the result of movement of free electrons i.e. these electrons which are not bound to the atoms of crystal lattice. In the figure e denotes one of free electrons moving with the velocity \vec{v}_e to the $+V$ potential. The hole conduction i.e. current of positively charged holes in the direction of $-V$ potential (dashed lines 1' and 2') results from hoppings of the valence electrons in the opposite direction (dashed lines 1 and 2).

B. Doped semiconductors

The above described mechanism of conducting electric current takes place in the intrinsic semiconductors i.e. undoped semiconductors. If in the crystal lattice some of the basic atoms were replaced by atoms of a different element we would say that the semiconductor is doped.

Let us assume that the basic element whose atoms create crystal lattice belongs to the IV group in the periodic table of elements, i.e. contains four valence electrons, like silicon (Si). Moreover let us assume that some atoms of basic element are replaced by the atoms of the element which belongs to the V group of periodic table i.e. by atoms which have five valence electrons, like e.g. arsenic (As). The atoms replacing basic atoms are called dopants or impurities. The dopants from higher group are called donors. In our case the arsenic is a donor which has one more electron than is needed to create bonds with four neighbouring basic atoms of silicon. This surplus valence electron is weakly bound to the dopant so it can be easily separated from the dopant and become free in the crystal lattice. The bounding energy of this electron is called the activation energy of donors and is denoted as E_d in the fig. 2b. This energy is substantially smaller than the energy gap E_g so for doped semiconductors much lower thermal energy is sufficient to obtain free electrons in the conduction band than for the intrinsic semiconductors. This is the reason why doped semiconductors at low temperatures have a few orders of magnitude higher conductivity than intrinsic semiconductors. The doped semiconductors which contain donors are called n-type semiconductors. The letter "n" comes from the word "negative" because the majority carriers of charge in this case are free electrons which have negative charge.

Now let us assume that the crystal lattice made of element of the IV group of periodic table (e.g. Si) is doped with atoms of the element belonging to the III group which has only three valence electrons (e.g. Ga). In this case even at low temperatures there is lack of electrons in the valence band so positively charged holes appear which are the majority carriers. This is why

such semiconductors are called p-type. Dopants of this kind are called acceptors. The process of creating holes can be explained on the graph representing band structure of p-type semiconductor (fig.2c). Electrons from valence band may be excited to the acceptor level so in valence band the holes are created which can conduct the current. Due to low activation energy of acceptors E_a which is much smaller than energy gap E_g even at low temperatures in the p-type semiconductors the concentration of holes and conductivity are much higher than in the intrinsic semiconductors.

To summarize what was said about semiconductors let us explain the dependence of conductivity of doped semiconductor on temperature (fig. 5). It is worth noticing that on the horizontal axis is the reciprocal of the absolute temperature so the temperature increases in the left direction. We remember also that the conductivity can be expressed by the equation (2). In the range of low temperatures (A→B segment of curve) the increase in temperature causes the increase in concentration of carriers of charge coming from dopants which results in the increase in conductivity of the semiconductor. At the point B the temperature is high

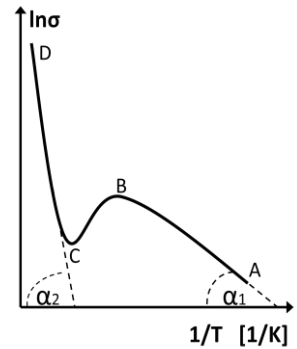


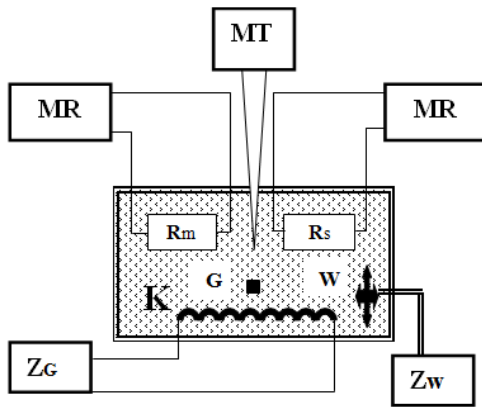
Fig.5. The dependence of conductivity of semiconductor on reciprocal of temperature.

enough to ionize all dopants. It means that for example in the case of n-type semiconductor from each dopant atom one surplus electron broke free which came to the conduction band. So the further increase in temperature (B→C) does not cause any increase in concentration of free carriers coming from dopants, that is why in this range the concentration remains constant and conductivity decreases with increasing temperature because mobility of free carriers μ decreases. The reason for this is the same as in metals, namely, atoms oscillate with bigger amplitude so the scattering of carriers is more effective. Above a certain high temperature (point C), comparable to the room temperature, but different for different semiconductors, the process of exiting electrons from valence band to the conduction band begins. In this region (C→D) conductivity depends on temperature exactly in the same way as in the intrinsic semiconductors, so conductivity in this region is called intrinsic conductivity. The increase in temperature causes very strong exponential increase in concentration of free electrons and holes, so although the mobility still slightly decreases the increase in concentration predominates which results in a very strong exponential increase in conductivity. In this range the carriers from dopants do not play any significant role and may be neglected so the dependence of resistance on temperature may be express by the equation (4). In the exercise our measurements will be conducted in the region of intrinsic conductivity (C→D).

The doped semiconductors are very important because they are widely applied in everyday life. Transistors and diodes made of semiconductors are the foundation of functioning of all domestic electronic facilities. For example light emitting diodes (LEDs) are applied in the screens of TVs and monitors or in ecologically friendly lamps which consume less energy than even efficient compact fluorescent lamps. Semiconductor lasers are used for saving and reading data on CDs and DVDs. Different kinds of semiconductor thermometers base on the strong dependence of their conductivity on temperature.

2. Measurement setup and experimental procedure

The experimental setup is presented in fig. 6. The measurement chamber contains the investigated metallic sample of resistance R_m and the semiconductor sample of resistance R_s . Moreover there is a thermometer MT, a heater G and a ventilator cooler W inside the chamber. The setup enables the measurements of resistance while heating the samples and while cooling them. The measurement should be conducted in the range of temperature from 20°C to 90°C.



- MR - ohmmeter
- MT - thermometer with thermocouple
- K - measurement chamber with samples
- R_m - metallic resistor
- R_s - semiconductor
- G - heater
- W - ventilator cooler
- Z_G - power supply of the heater
- Z_w - power supply of the ventilator cooler

Fig. 6. The scheme of the measurement setup

After connecting the electric circuit in accordance with the scheme (fig. 6) and after checking the circuit by the supervisor, turn on the ohmmeters. Read the initial resistances of the samples at the room temperature. Turn on the power supply of the heater and increase gradually the temperature of the samples up to 90°C by increasing the voltage across the heater. Read the temperature and resistance in 5°C steps. After reaching 90°C , at supervisor's request, repeat the measurements while cooling the samples. The detailed description of the measurements is included in the *Measurement Procedure* for this exercise which can be found on the website of the Laboratory of General Physics of our university.

3. Analysis of the data

On the basis of the measured data plot the resistance of metal as a function of temperature $R_m = f(t)$. Applying the method of linear regression find out the coefficients of the straight line which is the best fit to the measured points and their uncertainties. Comparing the equation of the straight line to the equation for the resistance of metal (3) find out the temperature coefficient of resistance α and calculate its uncertainty.

For the semiconductor plot the dependence $\ln R_s = f(1000/T)$, where T is the temperature expressed in Kelvins. Applying the method of linear regression find out the slope of the straight line which is the best fit to the measured points and its uncertainty. Having this slope and using equation (4) find out the energy gap E_g of the semiconductor and its uncertainty.

The detailed description of finding the energy gap and the temperature coefficient of resistance α and their uncertainties is included in the *Measurement Procedure* of this exercise.

4. Questions:

1. Describe the mechanism of conducting electric current in metals.
2. Explain the basic mechanisms of scattering free electrons in metals.
3. Present the dependence of resistance of metal on temperature (plot and equation). Explain the meaning of the temperature coefficient of resistance.
4. Describe the band structure of semiconductors. Explain the meaning of the energy gap.
5. Describe the mechanisms of conducting current in intrinsic and doped semiconductors.
6. Plot the dependence $\ln R_s = f(1000/T)$ for doped semiconductor and explain it.
7. What features of semiconductors are being changed by doping.
8. Give a few examples of applications of semiconductors.
9. How can you find out the temperature coefficient of resistance α and energy gap E_g .
10. Describe the methods of analysis of uncertainties of the measured and calculated quantities.

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