



**EXERCISE
57C**

INVESTIGATION OF HALL EFFECT

Key words: Hall effect, Hall voltage, Hall coefficient / constant, concentration of charge carriers, conductivity of electrons and holes,

Hall effect is a phenomenon where a potential difference (called a Hall voltage U_H) is generated in a conducting plate through which flows a current I when it is placed in an external magnetic field of induction B . This voltage is built-up between the opposite edges of the plate in a direction perpendicular to both the direction of the current flow I and the direction of the magnetic field vector B . It is caused by Lorentz force F_L acting on electrically charged particles moving in a magnetic field. The basic setup for Hall effect is shown in Fig. 1: A thin strip (of thickness d) of the material to be studied is placed in a magnetic field B oriented at the right angle to the strip.

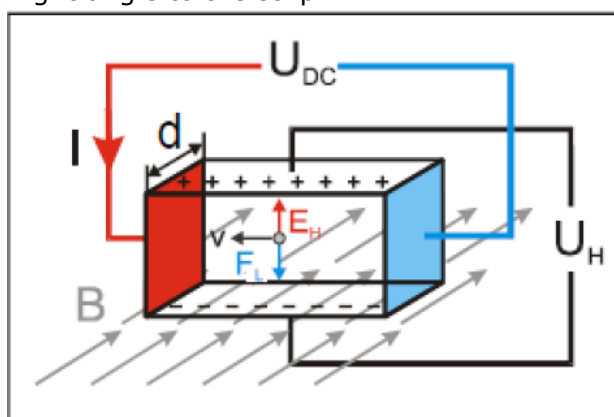


Fig. 1: Hall effect geometry: inside a conductor, which is located in the magnetic field B , the Lorentz force F_L induces the spatial separation of opposite charges, resulting in a Hall voltage U_H (I denotes a transverse current).

A current I is arranged to flow through the strip from left to right, and the voltage difference between the top and bottom is measured. Assuming the voltmeter probes are vertically aligned, the voltage difference is zero when $B = 0$.

The current I flows in response to an applied electric field, with its direction established by convention. However, on the microscopic scale, I is the result of either positive charges moving in the direction of I , or negative charges moving backwards. In either case, the magnetic Lorentz force $\mathbf{F}_L = q(\mathbf{v} \times \mathbf{B})$ causes the carriers to curve upwards. Since charge cannot leave the top or bottom of the strip, a vertical charge imbalance builds up in the strip. This charge imbalance produces a vertical electric field which counteracts the magnetic force, and a steady-state situation is reached. The vertical electric field can be measured as a transverse potential difference on the voltmeter.

Suppose now that the charge carriers are electrons ($q = -e$). In this case negative charge accumulates on the strip's top so the voltmeter would read a negative potential difference. Alternately, should the carriers be holes ($q = +e$) we measure a positive voltage.

As it was mentioned at the beginning, in this experiment, a conductor strip is located in a magnetic field B perpendicular to the direction of the current I , which generates a transverse electric field E_H . That field acts on charges with a force F_E , equal to:

$$F_E = q E_H$$

and directed opposite to Lorentz force. The flow of charges stops when the two forces obtain equilibrium, i.e. when $F_E = F_L$, which leads to the relation:

$$v B = E_H$$

If we assume uniform field distribution across the sample of width b , the generated Hall voltage U_H is connected with electric field E_H by:

$$E_H = \frac{U_H}{b}$$

resulting in an equation for Hall voltage U_H :

$$U_H = b v B$$

The average velocity of charges v may be related to current density j :

$$j = q n v$$

and thus to current I :

$$I = j S = q v n b d$$

where S is the cross-section area of the sample. We can calculate v from this equation and substitute it into the equation for Hall voltage, obtaining final relation for U_H :

$$U_H = \frac{1}{n e} \frac{B I}{d}$$

where: d – the thickness of a band-shaped conductor, n – the concentration of charge carriers, e – the elementary charge = 1.602×10^{-19} C. The factor $\frac{1}{ne}$, appearing in the above expression, is called the Hall constant R_H :

$$R_H = \frac{1}{n e}$$

The Hall constant gives a direct indication of the sign of the charge carriers; it is negative for electrons ($q = -e$) and positive for holes ($q = +e$). The sign of the Hall constant is determined by the polarity of the charge carriers. The Hall constant depends on the material and the temperature. For metals R_H is very small, however, for semiconductors R_H becomes significantly large. The concentration of charge carriers n can be determined experimentally by measuring the Hall voltage as a function of the magnetic field B for various currents.

The factor $\frac{1}{ned} = \gamma$ is the so called Hall probe constant. It depends on the material from which an investigated sample / strip was made.

Summarizing, the Hall effect is fundamental to solid-state physics and an important diagnostic tool for characterization of materials – particularly semiconductors. It provides a direct determination of both the sign of the charge carriers, e.g. electron or holes, and their density (concentration) in a given sample.