| EXERCISE 8 | DETERMINATION OF THE VISCOSITY OF LIQUID BY STOKES' LAW |
|---|---|

**Goals:** To study the motion of bodies falling in a liquid medium, to determine the viscosity coefficient of a liquid using Stokes' method and the Höppler viscometer.

**Problems:** The phenomenon of liquid viscosity. Stokes' Law, the motion of a ball in a viscous liquid.

## 1 Introduction

**Viscosity**, also known as internal friction, is the phenomenon where tangential forces oppose the movement of one part of a body relative to another. This phenomenon arises due to the thermal motion of molecules and intermolecular forces. As a result of the internal friction force acting between layers of liquid, a moving layer drags adjacent layers with it, with the speed being closer to its own, the more viscous the liquid is. Similarly, a stationary layer of liquid slows down the adjacent moving layers.

Since all real liquids are viscous, the phenomenon of viscosity plays a crucial role in fluid flow and the movement of a solid body in a liquid medium.

The fundamental method for describing fluid motion in hydrodynamics is the Euler method, which involves providing a relationship for the velocity vector **v** of the fluid flow at different points in space, depending on the coordinates of these points and time - $\mathbf{v} = f(\boldsymbol{r}, t)$.

A fluid flow is called **steady** or **stationary** if the velocity of the liquid at every point in the region occupied by the liquid does not change over time, i.e., **v** does not depend on time *t*.

The flow is called **laminar** or **layered** when the stream consists of layers moving relative to each other without mixing. At low flow velocities through a smooth pipe, the flow is laminar (the velocity at each point is uniquely determined) – see Fig. 1.

When the flow velocity of the liquid exceeds a certain critical value, characteristic of the given liquid, the motion ceases to be laminar. Mixing of different layers of the liquid occurs due to the formation of vortices. The velocity ceases to be a well-defined function of position coordinates. Such motion is called turbulent or vortex flow.

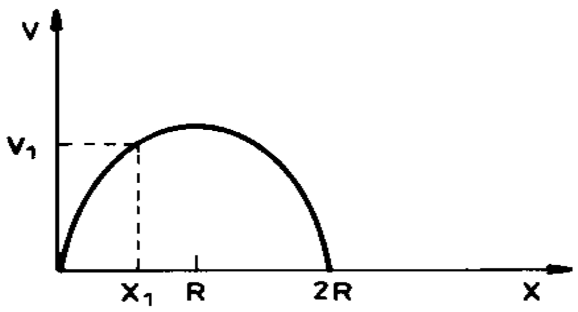


**Fig. 1.** Distribution of fluid velocity in a pipe with a circular cross-section; 2R - pipe diameter

The fluid in which there is no internal friction between the layers of the liquid, or it can be neglected, is called a perfect fluid. The empirical law describing the force of interaction occurring between two layers of liquid (laminar flow) was given by Newton. It can be expressed by the formula

$$|\boldsymbol{F}_t| = \eta S \left|\frac{dv}{dx}\right|. \qquad (1)$$

The value of the force $\boldsymbol{F}_t$ exerted by two adjacent layers of fluid on each other is proportional to the product of their contact area *S* and the velocity gradient $dv/dx$. The proportionality coefficient $\eta$ (eta) is called the viscosity coefficient. The unit of the viscosity coefficient in the SI system has the dimension $[\eta]$ = N s/m$^2$.

The viscosity coefficient of a medium depends on the temperature *T*. For liquids, the approximate relationship is valid

$$\eta = C e^{b/T}, \qquad (2)$$

where *C* and *b* are constants characteristic of the liquid, and *T* is the temperature on the Kelvin scale.

The phenomenon of viscosity, like diffusion and thermal conductivity, belongs to a group of phenomena collectively known as transport phenomena. In the case of viscosity, due to intermolecular interactions, momentum is transported between layers moving at different velocities. This transport helps to equalize the velocity throughout the entire flow of the liquid.

**Stokes' Law**

A solid body moving in a liquid medium encounters resistance. The mechanism of this phenomenon is as follows: the layer of liquid adhering to the surface of the moving body sets the remaining layers of liquid in motion. Thus, the viscosity of the liquid plays a crucial role. The resultant drag force acts in the opposite direction to the movement of the body. It has been experimentally determined that for low velocities, the value of the drag force $\boldsymbol{F}_t$ is directly proportional to the velocity $\mathbf{v}$, depends on the characteristic linear dimension of the body *l*, and on the viscosity coefficient $\eta$ of the liquid. The equation describing the drag force is as follows:

$$\boldsymbol{F}_t = -\alpha\, l\, \eta\, \mathbf{v}, \qquad (3)$$

where: $\alpha$ - constant depending on the body shape.

For a ball of radius *r* ($l = r$) the coefficient $\alpha = 6\pi$ and equation (3) turns into the so-called Stokes law

$$\boldsymbol{F}_t = -6\pi\, r\, \eta\, \mathbf{v}. \qquad (4)$$

**Free Fall of a Ball in a Liquid Considering Viscous Forces**

Let's consider the motion of a small ball with a radius *r*, freely falling in a viscous liquid. The forces acting on the ball are (see Fig. 2):

- the gravitational force on the ball $\boldsymbol{P} = m\boldsymbol{g} = \rho\, V\, \boldsymbol{g}$,
- the Archimedes buoyant force according $\boldsymbol{W} = -\rho'\, V\, \boldsymbol{g}$,
- the force of resistance to movement $\boldsymbol{F}_t = -6\pi\, r\, \eta\, \mathbf{v}$,

where: $V = {}^4/_3\, \pi r^3$ - the ball's volume, $\rho$ - the ball's material density, $\rho'$ - the liquid density.

The resultant force $\boldsymbol{F}$ acting on the body is given by

$$\boldsymbol{F} = \boldsymbol{P} + \boldsymbol{W} + \boldsymbol{F}_t. \qquad (5)$$

The value of the resultant force $\boldsymbol{F}$ decreases over time as the ball falls through the liquid. If the density of the material from which the ball is made is greater than the density of the liquid, then the motion of the ball freely falling in the liquid is accelerated, but not uniformly. This acceleration will decrease over time.

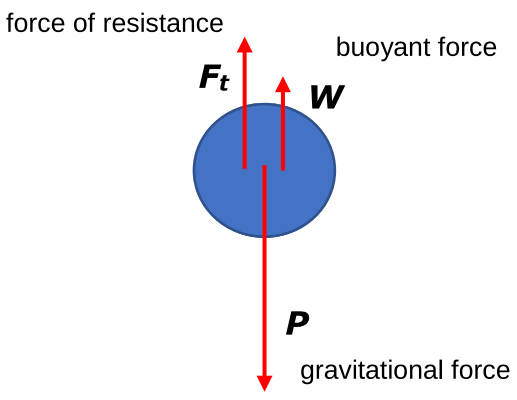


**Fig**. **2**. Forces acting on the ball during its descent in a viscous liquid.

The reason for the decrease in acceleration is the increasing velocity of the ball (see Fig. 3), which consequently leads to an increase in the drag force $\boldsymbol{F}_t$ associated with the liquid's viscosity.

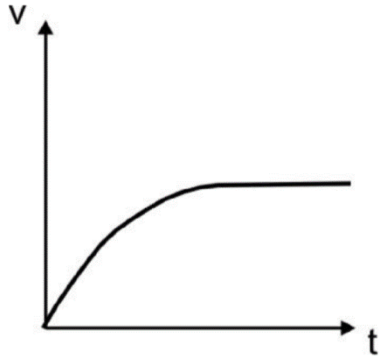


**Fig**. **3**. The velocity-time relationship for a ball starting to move in a viscous liquid with an initial velocity $v_0$=0.

After a sufficiently long time, the sum of the drag force and the buoyant force balances the gravitational force, at which point the resultant force ***F*** becomes zero. From that moment on, the ball moves at a constant speed known as the terminal velocity ($v_g$).

$$0 = \rho V g - \rho' V g - 6\pi r \eta v_g \qquad (6)$$

Using this formula, the viscosity coefficient of the liquid can be determined

$$\eta = \frac{2r^2 g(\rho - \rho')}{9\upsilon_g}. \qquad (7)$$

In the case of the motion of a large ball (with $r \approx R$) in a cylindrical vessel with a radius $R$, after accounting for corrections due to the influence of the side walls on the ball's motion, the formula for the viscosity coefficient can be written in the general form:

$$\eta = k \cdot (\rho_k - \rho_c) \cdot t \qquad (8)$$

where: $k$ is a constant, $t$ is the time taken to travel a given distance with uniform motion.

## 2 Measurement Principle and Measurement Setup

2.1. In the first part of the experiment, the viscosity coefficient is determined using Stokes' method, employing a wide glass cylindrical vessel filled with the liquid under investigation. On the outer side of the vessel, there are two adjustable rings (see Fig. 4). These rings are used to set the distance (*h*) that a small ball must travel in the liquid with uniform motion. The selected ball is released just above the surface of the liquid in such a way that its path approximately aligns with the axis of the vessel. The time (*t*) taken by the ball to travel between the rings is measured. The viscosity coefficient of the liquid is then determined using formula (7), taking into account that $v_g = h/t$.

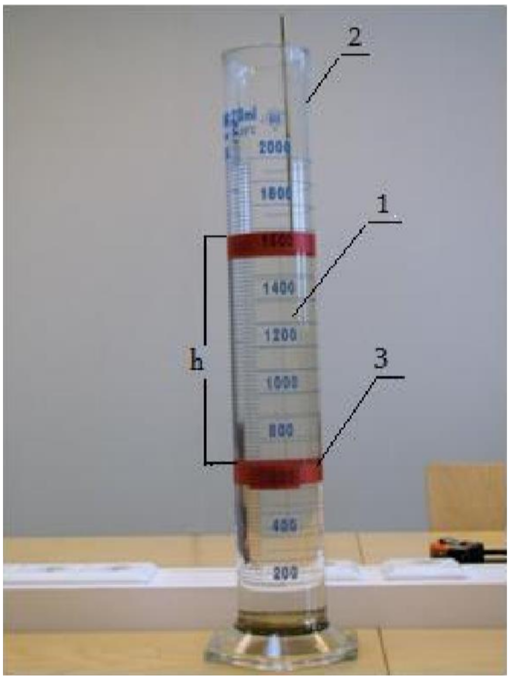


**Fig**. **4**. Device for measurement liquid viscosity by using Stokes' method:

1 – liquid
2 – glass cylinder
3 – rings
*h* – distance between rings

2.2. In the second part of the experiment, the viscosity coefficient of the liquid is determined using Stokes' method with a Höppler viscometer (see Fig. 5).

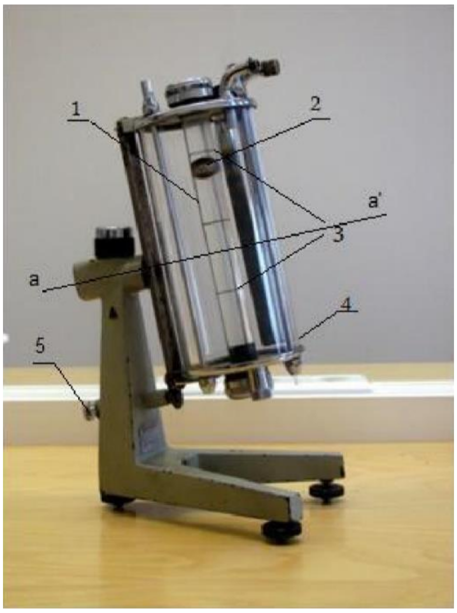


**Fig**. **5**. Höppler's viscometer:

1 – tube
2 – ball
3 – marks, between which the time of falling ball is measured
4 – thermostatic shield
5 – blocking device

A relatively large ball ($r \approx R$) moves in a liquid enclosed in a glass tube. The entire setup is housed within a thermostatic shield. The tube can rotate around the axis a-a'. The locking device allows the tube to be securely positioned. The measurement path is defined by the marks on the tube. The density of the liquid is provided. By measuring the time it takes for the ball to travel between the marks, the viscosity coefficient of the liquid is determined based on formula (8).

## 3 Tasks to be Completed

**A) Measurements:**

a) In a wide cylindrical vessel

1. Use a ruler with millimeter markings to measure the distance h between the rings.
2. Select several balls and measure their diameters d using a micrometer screw. Measure the diameter of each ball several times in different directions.
3. Determine the density ρc of the liquid by using an aerometer.
4. Measure the time *t* it takes for the balls to travel between the rings (upper and lower) using a stopwatch. Perform several measurements for each ball.

b) In the Höppler viscometer

1. For a given ball enclosed in the tube with the liquid under investigation, measure the time *t* it takes for the ball to fall between the marked lines using a stopwatch. To start the movement, unlock the tube with the thermostatic shield and rotate it around the axis a – a' by 180°.

**B) Data Processing:**

a) For the wide cylindrical vessel

1. Calculate the average diameter $\bar{d}$ and its measurement uncertainty u($\bar{d}$).
2. Calculate the average fall time $\bar{t}$ of the balls between the rings and its measurement uncertainty u($\bar{t}$).
3. Calculate the average mass $\bar{m}$ of the balls and its measurement uncertainty u($\bar{m}$).
4. Calculate the density $\rho_k$ of the ball and its uncertainty $u_c(\rho_k)$.
5. Calculate the viscosity coefficient $\eta$ of the liquid using the formula:

$$\eta = \frac{d^2 \cdot g \cdot t \cdot (\rho_k - \rho_c)}{18h} \quad (9)$$

   and its measurement uncertainty $u_c(\eta)$.

6. Determine the average value $\bar{\eta}$ from all the measurements.

b) For the Höppler viscometer

1. Calculate the average fall time $\bar{t}$ of the ball and its uncertainty u($\bar{t}$).
2. Calculate the viscosity coefficient $\eta$ of the liquid under investigation for the given measurement temperature using formula (8) and its uncertainty $u_c(\eta)$.

Data required for calculations:

- for the viscometer with a glass ball:

$k$ = 0,7941 · $10^{-6}$ $m^2/s^2$
$\rho_k$ = (2,41 ± 0,01) $g/cm^3$
$\rho_c$ = (1,261 ± 0,005) $g/cm^3$

- for the viscometer with a metal ball:

k = 0,1216 · $10^{-6}$ $m^2/s^2$
$\rho_k$ = (8,12 ± 0,01) $g/cm^3$
$\rho_c$ = (1,261 ± 0,005) $g/cm^3$

## 4 Questions:

1. What is the phenomenon of liquid viscosity?
2. Formulate and express Stokes' law using a formula.
3. What forces act on a ball falling in a viscous liquid?
4. Why does the ball eventually move with uniform motion after sufficient time?
5. How can the viscosity coefficient be determined?

prepared by: Anna Sycińska
Ewa Rysiakiewicz-Pasek
translation by: Liudmyla Filevska